Old and new trends in Hausdorff operators

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Abstract

Hausdorff operators (Hausdorff summability methods) appeared long ago aiming to solve certain classical problems in analysis. Modern theory of Hausdorff operators started with the work of Siskakis in complex analysis setting and with the work of Georgakis and Móricz-Liflyand in the Fourier transform setting. While the study of Hausdorff operators for power series in several dimensions has just begun, most interesting results for the Hausdorff operators of Fourier integrals are multi-variate. One of the most general definitions of the Hausdorff operator reads as

$$(\mathcal{H}f)(x) = (\mathcal{H}_\Phi f)(x) = (\mathcal{H}_{\Phi,A} f)(x) = \int_{\mathbb{R}^n} \Phi(u) f\big(xA(u)\big)\, du,$$

where $A = A(u) = (a_{ij})_{i,j=1}^n = \big(a_{ij}(u)\big)_{i,j=1}^n$ is the $n \times n$ matrix with the entries $a_{ij}(u)$ being measurable functions of u. This matrix may be singular on a set of measure zero at most; $xA(u)$ is the row n-vector obtained by multiplying the row n-vector x by the matrix A.

We give a brief overview of Hausdorff operators in various settings. Recent results in which conditions on the couple (Φ, A) are found to provide the boundedness of the operator in the real Hardy space are discussed. The case of product Hardy spaces is also studied. The whole stuff aims to clarify whether a new complete characterization of various Hardy spaces by means of Hausdorff operators is possible. Many open problems in the subject are formulated.