Limits of multipole pluricomplex Green functions

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ABSTRACT: Let Ω be a bounded hyperconvex domain in $\mathbb{C}^n$, $0 \in \Omega$, and S_ε a family of N poles in Ω, all tending to 0 as ε tends to 0. To each S_ε we associate the vanishing ideal I_ε and a pluricomplex Green function $G_\varepsilon = G_{I_\varepsilon}$, the pluricomplex Green function of the ideal I_ε.

Suppose that, as ε tends to 0, $(I_\varepsilon)_\varepsilon$ converges to I, (local uniform convergence), and that $(G_\varepsilon)_\varepsilon$ converges to G, in L^1_{loc}; then $G \geq G_I$.

If the limit ideal I is a complete intersection (same number of generators as the dimension), then convergence occurs and furthermore $G = G_I$. Conversely, if the Hilbert-Samuel multiplicity of I is strictly larger than its length (codimension, equal to N here), then $G > G_I$.

We work out the case of three poles in the bidisk.