SEMINARIS D'ANÀLISI MATEMÀTICA UAB-UB 2005–2006

- **Lawrence Narici (St John's University, NY, USA)**
  "The Hahn-Banach Theorem : history and applications".
  Dia i hora: dilluns 12 de setembre a les 15.00 h.
  Lloc: Aula C1/-128 de la UAB.
  Abstract: I love the Hahn-Banach theorem. I love it the way I love Casablanca and the Fontana di Trevi. It is something not so much to be read, as fondled. What is the Hahn-Banach theorem? Let f be a continuous linear functional defined on a subspace M of a normed space X. Take as the Hahn-Banach theorem the property that f can be extended to a continuous linear functional on X without changing its norm. Innocent enough, but the ramifications of the theorem pervade functional analysis and other disciplines (even thermodynamics!) as well. Where did it come from? Were Hahn and Banach the discoverers? The axiom of choice implies it, but what about the converse? Is Hahn-Banach equivalent to the axiom of choice? (No.) Are Hahn-Banach extensions ever unique? They are in more cases than you might think, when the unit ball of the dual is round, as for l_p with $1 < p < \infty$, for example, but not for l_1 or l_∞. Instead of a linear functional, suppose we consider a continuous linear map $A : M \to Y$ where Y is a normed space. Can A be continuously extended to X with the same norm? Well, sometimes. It depends principally on the range Y; more specifically, on the geometry of Y.

- **Peter Duren (Ann Arbor, Michigan)**
  "Schwarzian derivatives and univalence of analytic and harmonic functions".
  Dia i hora: dijous 29 de setembre, a les 15.00 h.
  Lloc: aula T2 de la UB.
  Abstract: The Schwarzian derivative of an analytic function is defined by $Sf = (f''/f')' - (1/2)(f''/f')^2$. Because of its Moebius invariance, it plays an important role in classical work on conformal mapping and differential equations. Around 1950, Nehari discovered that certain bounds on the Schwarzian provide criteria for univalence. In recent work with Martin Chuaqui and Brad Osgood, we have introduced a notion of Schwarzian derivative for an "arbitrary" complex-valued harmonic function. The generalization preserves many of the classical properties, including univalence criteria. In this context it is natural to identify a harmonic mapping with its

Weierstrass-Enneper lift to a minimal surface.

- **Konstantin Dyakonov (UB)**
  "Self-improving behaviour of inner functions as multipliers".
  Dia i hora: dilluns 3 d'octubre, a les 15.00 h.
  Lloc: aula T2 de la UB.
  Abstract: For an inner function I, it is often true that multiplication by I either destroys smoothness completely or does not affect it at all. We shall discuss this phenomenon in various concrete situations.

- **Hartmut Führ (Zentrum Mathematik, Munich)**
  "Shannon's sampling theorem for the Heisenberg group".
  Dia i hora: 10 d'octubre a les 15.00 h.
  Lloc: aula petita del CRM.
  Abstract: I present a version of Shannon's sampling theorem for Paley-Wiener spaces associated to the spectrum of the sub-Laplacian on the simply connected Heisenberg Lie group. While the proof technique works for arbitrary stratified Lie groups, I will stick to the case of the Heisenberg group in order to keep the exposition simple. Moreover, the results for this group lead to interesting new facts in Gabor analysis.

- **Paco Villarroya (Univ. de València i UCLA)**
  "Condicions d'afitació per a formes multilineals associades a nuclis de Calderón- Zygmund".
  Dia i hora: 17 d'octubre a les 15.00 h.
  Lloc: aula C1/-128 de la UAB.
  Abstract: Mitjançant un resultat de tipus T(1) es proven condicions necessàries i suficients per a l'afitació d'operadors bilineals associats a variables de Calderón-Zygmund. El resultat inclou generalitzacions de la Transformada de Hilbert bilineal amb nucli variable.

- **Hidenobu Yoshida (Chiba University, Japó)**
  "Potential-theoretic exceptional sets at infinity on a cone".
  Dia i hora: 24 d'octubre a les 15.00 h.
  Lloc: aula T2 de la UB.
  Abstract: (joint work with I.Miyamoto) By following Lelong-Ferrand (1949), in 1980 Essén and Jackson investigated boundary behaviors near infinity outside a "minimally thin set" for a positive superharmonic function defined on a half-space and described a quatitative size of minimally thin sets. In 1981 Aikawa soon introduced the notion of an "a-minimally thin set ($0 < a \leq 1$)" which classfies a minimally thin set (= a 1-minimally thin set), and extended their results. Then Essén, Jackson and Ripp on followed him in 1984.

All works stated above are always connected with a half-space which is like a ball having infinity on the boundary and one of cones extending to infinity. A cone extending to infinity has generally a wedge at infinity and in this respect is different from a half space.

In this talk, we shall show how some results in their works stated above are extended to a positive superharmonic functions defined on a cone extending to infinity.

- **Jim Wright (University of Edinburgh)**
  "A class of product BMO functions".
  Dia i hora: 24 d'octubre a les 16.00 h.
  Lloc: aula T2 de la UB.
  Abstract: To understand the geometric properties of product BMO functions seems difficult. In fact to construct explicit examples is not straightforward. Via a Hardy type inequality for prodcut H1 functions, we present a class of product BMO functions which may be of some interest.

- **Jean-Pierre Rosay (University of Wisconsin, Madison)**
  "Polynomial convexity of arcs, many years later, and still with unsatisfactory progress´´".
  Dia i hora: 7 de novembre a les 15.00 h.
  Lloc: aula T2 de la UB.
  Abstract: After many years things sometimes simplify a lot. An example is Rossi's local maximum principle whose proof is a 2 line exercise. It is probably naive to expect that polynomially convexity of arcs will ever become a trivial result. Still it is good to re-visit this question, hoping to get a better result and also an easier proof. I had only a very limited success in a simplification of the proof of polynomial convexity.

- **Maksim Ya. Mazalov (Military University of Air Defence Forces of Russia, Smolensk)**
  "Uniform approximation theorem for the square of the Cauchy-Riemann operator".
  Dia i hora: 14 de novembre a les 15.00 h.
  Lloc: aula petita del CRM.
  Abstract: Let X be an arbitrary compact set of the complex plane and $\bar{\partial}$ be the Cauchy-Riemann operator. It is proved that every function f continuous on X and satisfying the equation $\bar{\partial}^2 f = 0$ on the interior of X can be approximated uniformly on X by functions satisfying the same equation in some neighborhoods of X.

  This problem was proposed by Joan Verdera as a conjecture.

- **Franciso Hernández (Universidad Complutense)**
  "Singularidad estricta y compacidad debil de inclusiones en $L^1 + L^\infty$".
  Dia i hora: 21 de novembre a les 15.00 h.
  Lloc: aula T2 de la UB.
  Abstract: Se estudia la singularidad estricta y la compacidad debil de operadores inclusion entre espacios de funciones invariantes por reordenamiento sobre $[0, \infty)$. Se analizan las inclusiones extremas a izquierda $L^1 \cap L^\infty \hookrightarrow E$, y a la derecha $E \hookrightarrow L^1 + L^\infty$. Mostraremos como la escala de los espacios L^p-debiles separables W^p, $(1 < p < \infty)$ son la "frontera" para la singularidad de inclusiones $E \hookrightarrow L^1 + L^\infty$.

- **Severine Rigot (Université de Paris-Sud)**
  "Besicovitch covering property in Carnot groups".
  Dia i hora: 21 de novembre a les 16.00 h.
  Lloc: aula T2 de la UB.
  Abstract: Covering theorems are known to be useful tools in measure theory. They reflect the geometry of the space and can for instance be used to establish connections between local and global behaviour of measures. The so called Besicovitch covering property turns indeed out to be equivalent to the validity of a theorem of differentiation of measures for all locally finite Borel measures. It originates from Besicovitch who proved its validity for the Euclidean space. We will present in this talk a counterexample to this covering property in Carnot groups equipped with a Carnot-Caratheodory distance. This also illustrates once more the nowadays well-known fact that subriemannian and Euclidean geometry can be extremely different.

- **Carlos Pérez (Universidad de Sevilla)**
  "El Teorema de extrapolación de Rubio de Francia: una nueva demostración´´.
  Dia i hora: dimarts 22 de novembre a les 15.00 h.
  Lloc: aula B2 de la UB.
  Abstract: El Teorema de extrapolación de J. Luis Rubio de Francia es uno de los teoremas más importantes y más bellos del Análisis Armónico moderno. Es una herramienta muy útil pues proporciona un medio muy eficaz para obtener acotación de operadores en los espacios L^p a partir de la informacion contenida en el caso L^2.

  El propósito de esta charla es el de dar una demostración directa y relativamente elemental de este Teorema. Tiene la ventaja de que se puede extender a muchos mas espacios que surgen de forman natural en el Ánalisis como los espacios invariantes por reordenadas.

- **Daniel Faraco (UAM)**
  "Funciones de distorsión acotada y finita en el espacio".
  Dia i hora: 28 de novembre, a les 15.00 h.
  Lloc: CRM.
  Abstract: En el plano, existe una rica teoría de funciones de distorsión acotada (funciones cuasiregulares) debido a que gracias a la ecuación de Beltrami se pueden emplear técnicas de variable compleja. Sin embargo en dimensiones mayores el correspondiente sistema no se comporta, ni mucho menos, de la misma manera y los métodos para entender estas funciones deben ser distintos. En la charla indicaremos varias de las propiedades de estas funciones, haciendo especial incapie en los asuntos de integrabilidad mejorada de fundamental importancia en la teoría de ecuaciones elípticas. En particular, daremos pruebas sencillas de los teoremas clásicos de Gehring e Iwaniec sobre este tema e indicaremos como estas nuevas pruebas si extienden al caso de funciones de distorsión finita donde la distorsión se supone exponencialmente integrable. Estos resultados permiten entender como estas funciones afectan al area, a la dimensión de Hausdorff y que tamaño tienen los conjuntos evitables.

- **Eva Gallardo (Universidad de Zaragoza)**
  "Operadores de composición en espacios de Hardy de un dominio simplemente conexo".
  Dia i hora: 12 de desembre, a les 15.00 h.
  Lloc: aula petita del CRM.
  Abstract: En esta charla, estudiaremos la continuidad y compacidad de los operadores de composición C_Φ en los espacios de Hardy $H^p(\Omega)$, donde Ω es un dominio simplemente conexo. Mostraremos que la geometría del dominio Ω juega un papel fundamental en dicho estudio determinando, en cierto modo, el Principio de Subordinación de Littlewood para C_Φ en $H^p(\Omega)$. En particular, daremos respuesta a una cuestión planteada por Shapiro y Smith y caracterizaremos la continuidad y compacidad de C_Φ en $H^p(\Omega)$ cuando Ω es un dominio de Lavrentiev.
  (Trabajo en colaboración con María J. González y Artur Nicolau).

- **Dmitry Jakobson (McGill University, Canada)**
  "Lower bounds for spectral function of the Laplacian and for the remainder in local Weyl law".
  Dia i hora: 12 de desembre, a les 16.00 h.
  Lloc: aula petita del CRM.
  Abstract: Weyl's law gives the main term for the asymptotic distribution of eigenvalues of Laplacians on compact Riemannian manifolds. In this talk, we shall be concerned with lower bounds for the spectral function of the Laplacian and for the pointwise error

term in Weyl's law. On a flat square 2-torus, counting eigenvalues is equivalent to counting lattice points inside a circle. We show a simple approach which proves (on a general Riemannian manifold) a pointwise analogue of lower bounds of Hardy and Landau in the classical Gauss circle problem. We shall explain how the techniques of thermodynamic formalism for hyperbolic flows yield stronger estimates on negatively curved manifolds. This is joint work with I. Polterovich (Univ. de Montreal).

- **Jordi Pau (UB)**
  "Capacitats i conjunts de determinació en espais de tipus Dirichlet".
  Dia i hora: 19 de desembre, a les 15.00 h.
  Lloc: UB.
  Abstract: Sigui A una classe de funcions holomorfes (o harmòniques) en un domini Ω del pla (o de $\mathbb{R}^N$). Diem que un subconjunt $E \subset \Omega$ és un conjunt de determinació per A si $\sup_{z\in E} |u(z)| = \sup_{z\in\Omega} |u(z)|$ per a tota funció $u \in A$. En aquesta xerrada estudiarem caracteritzacions del conjunts de determinació pels espais locals de Dirichlet $D(\mu)$ harmònics i pels espais de Dirichlet harmònics amb pesos. La caracterització vé donada en termes de capacitats associades a aquests espais.

- **Joaquim Bruna (UAB)**
  "Generadors per translacions dels Lp(R) i densitats".
  Dia i hora: 9 de gener, a les 15.00 h.
  Lloc: CRM.
  Abstract: Continuant amb el tema d'un seminari de fa un parell d'anys, relacionarem les situacions seguents: a) parelles (f,E) de funcions f i conjunts E tals que les E-transladades de f generen un espai de Lebesgue Lp(R) b) conjunts d'unicitat per a classes quasianalítiques c) densitats del tipus Polya.

  També analitzarem la situació quan es consideren transladades afins, es a dir, quan E inclou parametres de translació i també de dilatació, i mencionarem algun problema interessant que apareix.

- **Alexei Poltoratski (Texas A & M University)**
  "Toeplitz Kernels and the Uncertainty Principle".
  Dia i hora: 16 de gener, a les 15.00 h.
  Lloc: aula petita del CRM.
  Abstract: One of the classical problems of harmonic analysis is the problem of completeness of exponential functions in L^2 spaces. A beautiful solution to this problem was found by Beurling and Malliavin in 1960's. In my talk I will discuss a joint project with N.

Makarov (Caltech) where we attempt to find extensions and applications of the Beurling-Malliavin theory.

- **María Jesús Carro (UB)**
  "Acotaciones de tipo débil restringido".
  Dia i hora: 23 de gener, a les 15.00 h.
  Lloc: aula T2 de la UB.
  Abstract: El objetivo de este seminario es hacer una revisión de algunos de los últimos resultados sobre acotación de operadores de los cuales se conoce una acotación de tipo débil sobre conjuntos característicos. La motivación principal del problema que plantearemos es el de la convergencia en casi todo punto de la serie de Fourier de una función en el espacio $L \log L$, aún sin resolver.

- **Xiao Zhong (Jyvaskyla)**
  "A lemma on the higher integrability of functions and its applications".
  Dia i hora: 30 de gener.
  Lloc: aula petita del CRM.
  Abstract: I will discuss a higher integrability lemma of Gehring type, and its applications in Geometric Function Theory and in PDEs. The talk is based on joint work with D. Faraco and P. Koskela, and joint work with M. Bildhauer and M. Fuchs.

- **Javier Parcet (CRM)**
  "Nociones de Análisis Armónico No Conmutativo I".
  Dia i hora: 6 de febrer, a les 15.00 h.
  Lloc: aula T2 de la UB.
  Abstract: Desde los orígenes de la Mecánica Cuántica, el proceso de sustituir funciones por operadores (cuantización) ha tenido un gran impacto en las matemáticas del siglo XX. Los trabajos de von Neumann o Connes han dado lugar por ejemplo a la cuantización de la Teoría de la Medida o la Geometría Diferencial. La consecuencia principal es la pérdida de conmutatividad para el producto.

  El Análisis Armónico No Conmutativo se ha identificado históricamente con el estudio de la transformada de Fourier/representaciones unitarias sobre grupos topológicos no conmutativos. No obstante, la teoría se ha desarrollado de forma espectacular en los últimos años y se abordan hoy por hoy una gran variedad de problemas. Existen así formulaciones no conmutativas de multiplicadores de Fourier, operadores maximales, procesos p-estables, sumas de variables aleatorias independientes, descomposiciones de tipo Calderón-Zygmund, teoremas ergódicos maximales, desigualdades de martingalas... Este significativo desarrollo se apoya en nuevas y profundas teorías como la Probabilidad Libre/Cuántica y los Espacios de Operadores.

Todas estas herramientas han hecho posible estudiar (desde el punto de vista no conmutativo) la interacción entre Análisis Armónico, Probabilidad y Geometría de Espacios de Banach, desarrollada básicamente en los años 60 y 70 por los trabajos de Burkholder, Fefferman, Gundy, Pisier o Rosenthal entre otros. Nuestro objetivo es hacer un esbozo del estado actual de la teoría sin ánimo de completitud y orientado a una audiencia sin experiencia en el contexto no conmutativo.

- **Javier Parcet (CRM)**
  "Nociones de Análisis Armónico No Conmutativo II".
  Dia i hora: 9 de febrer, a les 12.00 h.
  Lloc: aula petita del CRM.

- **Andreas Hartmann (Burdeus)**
  "Interpolation in big Hardy-Orlicz spaces, or how to split a separated Blaschke sequence".
  Dia i hora: 13 de febrer, a les 15.00 h.
  Lloc: aula T2 de la UB.
  Abstract: Free interpolation in Hardy spaces is caracterized by the well-known Carleson condition. The result extends to Hardy-Orlicz spaces subjected to some conditions forcing these spaces to stay within the scale of classical Hardy spaces. For the Smirnov and the Nevanlinna classes, interpolating sequences have been characterized recently in terms of existence of (quasi-bounded) harmonic majorants. Since the Smirnov class can be seen as the union over all Hardy-Orlicz spaces associated with a so-called strongly convex function, it is natural to ask what happens between the scale of classical Hardy spaces H^p, $p > 0$, and the Smirnov class. The aim of this talk is to characterize interpolating sequences for "big" Hardy-Orlicz spaces, more precisely for a class of Hardy-Orlicz spaces who carry an algebraic structure.

- **José Calabuig (Universitat Politècnica de València)**
  "El Teorema de Bennet-Maurey-Nahoum para espacios de funciones integrables respecto de una medida vectorial".
  Dia i hora: 20 de febrer, a les 15.00 h.
  Lloc: aula petita del CRM.
  Abstract: Sea (Ω, Σ, μ) un espacio de medida finito. Dado un espacio de Banach X, se dice que una función $f : \Omega \to \mathbb{K}$ es integrable respecto de la medida vectorial $m : \Sigma \to X$ si (1) para todo $x' \in X'$ se tiene que $f \in L^1(\langle m, x' \rangle)$, (2) para cada $E \in \Sigma$ existe $x_E \in X$ tal que

$$\int_E f d\langle m, x' \rangle = \langle x_E, x' \rangle, \text{ para todo } x' \in X'.$$

El conjunto formado por las funciones que verifican (1) y (2) se denota por $L^1(m)$ y es un espacio de Banach dotado con la norma $\|f\|_{L^1(m)} = \|m_f\|$ donde $\|m_f\|$ es la semivariación de la medida asociada a f,

$$m_f : \Sigma \to X, \quad \text{dada por} \quad m_f(E) = x_E.$$

Demostraremos que $L^1(m)$ es isométricamente isomorfo con el espacio de los multiplicadores definidos en $L^{p'}(\mu)$ y con valores en $L^1(m)$ (dotado con la norma inducida por la norma de los operadores). Esta identificación nos permitirá, usando el Teorema de Maurey-Rossenthal, obtener una versión del Teorema de Bennet-Maurey-Nahoum en los espacios $L^1(m)$.

- **Alexander Tumanov (University of Illinois)**
  "Testing analyticity on families of curves".
  Dia i hora: 27 de febrer, a les 15.00 h.
  Lloc: aula T2 de la UB.
  Abstract: Let D be a domain in complex plane covered by a one parameter family of Jordan curves. Let f be a continuous function in D such that for every curve C of the family the restriction of f to C holomorphically extends inside C. Does this imply that f is holomorphic in D? There is a simple counterexample in which the curves are circles containing inside a fixed point. In the late 80s, Globevnik solved the question in the affirmative for a family obtained by rotating an arbitrary Jordan curve about a point in the exterior of the curve. Surprisingly, for a translation invariant family of circles, the question known as "strip-problem" has been open until 2002, when it was solved by the speaker. The general case, aslo called sometimes "strip-problem" is still open. There are partial results for arbitrary families of circles, for functions and families with additional regularity, and for equations more general, then the Cauchy-Riemann equations (Agranovsky, Ehrenpreis, Globevnik, Tumanov, etc.). Despite the one variable nature of the strip-problem, the methods involve several complex variables, in particular the extendibility of CR functions. I will present some of the results on the problem.

- **Daniel Suárez (UAB)**
  "Norma esencial y el álgebra de Toeplitz sobre espacios de Bergman".
  Dia i hora: 6 de març, a les 15.00 h.
  Lloc: aula petita del CRM.
  Abstract: Sea A^2 el espacio de Bergman sobre el disco unidad. El álgebra de Toeplitz $\mathfrak{T}$ se define como el álgebra cerrada generada por operadores de Toeplitz T_a, con $a \in L^\infty$, actuando sobre A^2. Veremos una estimación de la norma esencial para operadores en $\mathfrak{T}$, generalizaciones a otros espacios de Bergman, y como consecuencia,

una caracterización de compacidad en términos de la transformada de Berezin.

- **Albert Clop (UAB)**
  "Aplicacions quasiconformes i conjunts evitables".
  Dia i hora: 13 de març, a les 15.00 h.
  Lloc: aula T2 de la UB.
  Abstract: El problema de Painlevé (sobre singularitats evitables de funcions analítiques i acotades) pot ser plantejat també per a funcions quasiregulars. En aquesta xerrada parlarem d'aquestes funcions i de l'equació de Beltrami, i relacionarem el problema d'evitabilitat amb algunes qüestions de distorsió quasiconforme.

- **Nacho Monreal (UAB)**
  "Espectro de medias integrales para funciones de Bloch".
  Dia i hora: 20 de març, a les 15.00 h.
  Lloc: aula petita del CRM.
  Abstract: La conjetura de Brennan (1978) originalmente planteaba si $\int_{\mathbb{D}} |f'|^p dA < \infty$, con $-2 \leq p \leq \frac{2}{3}$, siendo $f : \mathbb{D} \longrightarrow \mathbb{C}$ una aplicación conforme. Actualmente el problema para $p = -2$ sigue abierto. Nos planteamos una cuestión similar: ¿Para qué valores de p se verifica que $\int_{\mathbb{D}} e^{p\Re g}\, dA < \infty$, donde g es una función de Bloch definida en el disco unidad $\mathbb{D}$? Daremos la solución bajo determinadas condiciones de la función g.

- **Zeev Ditzian (University of Alberta)**
  "Multivariate approximating averages in various spaces".
  Dia i hora: 27 de març, a les 15.00 h.
  Lloc: aula T2 de la UB.
  Abstract: For functions on the unit sphere S^{d-1} $(d \geq 3)$ we show that

$$\|S_\theta f - f\|_{L^p(S^{d-1})} \approx \inf \left(\|f - g\|_{L^p(S^{d-1})} + \theta^2 \|\tilde{\Delta} g\|_p \right), \quad 1 \leq p \leq \infty, \tag{0.1}$$

  where

$$S_\theta f(x) = \frac{1}{m_\theta} \int_{x \cdot y = \cos\theta} f(y) d\gamma(y), \quad S_\theta 1 = 1 \tag{0.2}$$

  and $\tilde{\Delta}$ is the Laplace-Beltrami operator. The improvement on previous results is that for those, $\sup_{|t| \leq \theta} \|S_t f - f\|_{L^p(S^{d-1})}$ was used on the left side of (0.1). It is important to note that for the circle T $(d = 2)$ (0.1) is no longer valid. Extensions to higher levels of smoothness and analogues for $\mathbb{R}^d$ $(d = 2)$ will be given. It will be shown that for $d \geq 3$ $S_\theta^m f$ for $m = 1, 2, \ldots$ are smoother than f and that the improvement is greater when the dimension is higher.

- **J.P. Rosay (University of Wisconsin, Madison)**
"Extension of holomorphic bundles to the disc, a Serre problem on Stein bundles".
Dia i hora: 3 d'abril, a les 14.00 h.
Lloc: aula B2 de la UB.
Abstract: We shall study (locally trivial) holomorphic bundles over open sets in $\mathbb{C}$ with fiber $\mathbb{C}^n$. We shall assume that the bundle is given by transition fiber automorphisms that are locally independent of the base point and that belong to a group G of automorphisms of $\mathbb{C}^n$. Our goal is to extend such bundles to bundles defined over larger open sets in $\mathbb{C}$ and in fact over the whole Riemann sphere. For the extended bundles the fiber automorphisms will depend on the base point. As a corollary of our results one gets examples of holomorphic bundles on the unit disc (and $\mathbb{C}$) with fiber C^2 and with gluing polynomial automorphisms that are non trivial and non Stein. This answers the last question in the Serre Problem on Stein bundles, left open after the counterexamples of Skoda, Demailly and Coere-Loeb.

- **Adela Moyua (Univ. País Basc)**
"Ecuación de Shrödinger en el caso periódico".
Dia i hora: 3 d'abril, a les 15.00 h.
Lloc: aula B2 de la UB.
Abstract: Si consideramos la ecuación de Schrödinger para el caso periódico

$$\begin{cases} 2\pi i \partial_t u = \partial_x^2 u, \\ u(x,0) = u_0(x) = \sum_j a_j e^{2\pi i j x}, \end{cases}$$

la solución viene definida por $u(x,t) = e^{it\partial_x^2}\, u_0 = \sum_k a_k e^{2\pi i (k^2 t + kx)}$.
Obtenemos unas estimaciones de tipo Strichartz, comenzando con un resultado bilineal:

Si los datos iniciales $f(x)$ y $g(x)$ verifican $d\,(\operatorname{supp} \hat{\mathrm{f}}, \operatorname{supp} \hat{\mathrm{g}}) = \mathrm{m}_0$, entonces

$$\left\| e^{it\partial_x^2}\, f\, \overline{e^{it\partial_x^2}\, g} \right\|_{L^2_{[0,\delta]} L^2_{[0,1]}} \leq (\Delta(m_0,\delta))^{1/2}\, \|f\|_{L^2}\, \|g\|_{L^2},$$

donde $\Delta(m_0,\delta) = \delta$ si $m_0 \geq \delta^{-1}$; $1/m_0$ si $\delta^{-1/2} < m_0 < \delta^{-1}$; $\delta^{1/2}$ si $m_0 \leq \delta^{-1/2}$ y como consecuencia probamos la siguiente estimación

$$\left\| e^{it\partial_x^2}\, u_0 \right\|_{L^4_{[0,1]\times I}} \leq |I|^{1/8}\, \|u_0\|_2.$$

Si se considera la correspondiente función maximal $\sup_t |u(x,t)|$, para dato inicial $u_0(x) = \sum_{|j|\leq N} a_j e^{2\pi i j x}$ obtenemos la siguiente estimación

$$\left\| \sup_{t\in[0,\delta]} |u(x,t)| \right\|_{L^4_x} \leq \delta^{1/4} N^{1/2} N^{\epsilon} \|u_0\|_2.$$

Utilizando estos resultados se prueba que la ecuación nolineal de Schrödinger está globalmente bien propuesta en el espacio $L^4([0,1]\times[0,1])$ para datos en L^2.

- **Luis Escauriaza (Univ. País Basc)**
"Continuación única cuantitativa en hiperplanos característicos de algunas ecuaciones parabólicas o dispersivas".
Dia i hora: 24 d'abril, a les 15.00 h.
Lloc: aula petita del CRM.
Abstract: Explicaré la cuantificación apropiada de algunos resultados de continuación única cualitativa. El primero consiste en la siguiente propiedad de continuación única retrógrada de L. Escauriaza, G. Seregin y V. Sverak:
Si $u : \mathbb{R}^n \setminus B_R \times [0,T] \longrightarrow \mathbb{R}$ *verifica en* $\mathbb{R}^n \setminus B_R \times [0,T]$ *las desigualdades*

$$|\triangle u + \partial_t u| \leq M\left(|u| + |\nabla u|\right) \ , \ |u(x,t)| \leq M e^{M|x|^2} \ ,$$

entonces, $u \equiv 0$ *en* $\mathbb{R}^n \setminus B_R \times [0,T]$*, si* $u(\,\cdot\,,0) \equiv 0$ *en* $\mathbb{R}^n \setminus B_R \times [0,T]$.
Los segundos están relacionados con otros resultados de continuación única de C.E. Kenig, L. Vega y G. Ponce para ecuaciones no lineales de tipo Schrödinger o KdV.

- **Enrico Laeng (Politecnico di Milano)**
"Evaluating the exact norm of some linear and sub-linear operators on L^p and on other function spaces. Recent results, remarks, and open problems".
Dia i hora: 8 de maig, a les 15.00 h.
Lloc: aula B1.
Abstract: The rearrangement inequality $SMf(x) \leq MSf(x)$ holds, where M is the uncentered Hardy-Littlewood maximal operator, S is the symmetric rearrangement operator, and $f : \mathbb{R} \to \mathbb{R}$. We completely characterize those functions f that yield equality for a.e. $x \in \mathbb{R}$. We apply this result to the computation of the exact norm of M on Lorentz and Marcinkiewicz spaces. The function $|x|^{-1/p}$ is extremal for M seen as an operator from weak-L^p into itself. The same function plays a special role in evaluating the exact norm of the Hilbert transform, from $L^p(\mathbb{R})$ into $L^p(\mathbb{R})$ for $1 < p < \infty$, and the norm of the Hilbert transform turns out to coincide with (or

be related to) the norm of some remarkable families of Fourier multipliers, probably including some kinds of discrete Hilbert transform.

- **Boris Mordukhovich (Wayne State University)**
  "Variational analysis and generalized differentiation: new trends and developments".
  Dia i hora: 8 de maig, a les 16.00 h.
  Lloc: aula B1.
  Abstract: Variational analysis has been recognized as a rapidly growing and fruitful area in mathematics concerning mainly the study of optimization and equilibrium problems, while also applying perturbation ideas and variational principles to a broad class of problems and situations that may be not of a variational nature. One of the most characteristic features of modern variational analysis is the intrinsic presence of nonsmoothness, which naturally enters not only through initial data of optimization-related problems but largely via variational principles and perturbation techniques applied to problems with even smooth data. This requires developing new forms of analysis that involve generalized differentiation.

  In this talk we discuss some new trends and developments in variational analysis and its applications mostly based on the author?s recent 2-volume book ?Variational Analysis and Generalized Differentiation, I: Basic Theory, II: Applications,? Springer, 2006. Applications particularly concern optimization and equilibrium problems, optimal control of ODEs and PDEs, mechanics, and economics. The talk does not require preliminary knowledge on the subject.

- **John Garnett (UCLA)**
  "Interpolating Blaschke Products and Approximations by Inner Functions".
  Dia i hora: 15 de maig, a les 15.00 h.
  Lloc: aula petita del CRM.
  Abstract: A survey of old problems and recent results related to the problem of approximating, uniformly, every H^∞ function with $|f(e^{i\theta}| = 1$ almost everywhere by Blaschke products whose zeros form H^∞ interpolation sequences.

- **Joan Verdera (UAB)**
  "Acotacions puntuals per la integral singular maximal".
  Dia i hora: 22 de maig, a les 15.00 h.
  Lloc: aula petita del CRM.
  Abstract: Resulta que es pot controlar $T * f$ directament per Tf en certes condicions. Aquí T és una integral singular homogènea amb nucli "smooth" i $T*$ és la integral singular maximal. El control fins i tot arriba a ser puntual. Hi ha desigualtats del tipus de Cotlar en

què el terme que conté Mf desapareix i només queda el terme que conté $M(Tf)$. Això dóna noves desigualtats L^p i de tipus feble fins i tot per les transformades de Riesz i de Beurling. Al final s'acaba treballant amb multiplicadors de Fourier i apareixen certes relacions algebriques que caracteritzen la validesa de les desigualtats puntuals en certs casos.

- **Sergey Tikhonov (CRM)**
  "Convergence of trigonometric series".
  Dia i hora: 29 de maig, a les 15.00 h.
  Lloc: aula petita del CRM.
  Abstract: We discuss three new convergence criteria (for $p = \infty$, $1 < p < \infty$, and $p = 1$) of belonging of sums of trigonometric series to L_p. One-dimensional and multi-dimensional cases are examined. We also study Hardy-Littlewood type theorems for multiple trigonometric and Walsh series in L_p with Muckenhoupt weights, in the Besov space, and in BMO.

- **V.I. Kolyada (U. de Karlstad)**
  "Gagliardo-Nirenberg-type inequalities and estimates of the moduli of continuity".
  Dia i hora: 12 de juny, a les 15.00 h.
  Lloc: aula T2 de la UB.
  Abstract: We study multiplicative inequalities of Gagliardo-Nirenberg type which connect partial moduli of continuity and partial derivatives of functions with respect to a fixed variable in diferent Lorentz norms.

- **Sergei Treil (Brown University)**
  "Operator corona theorem and geometry of holomorphic vector bundles".
  Dia i hora: 13 de juny, a les 12.00 h.
  Lloc: aula gran del CRM.
  Abstract: The classical Carleson Corona Theorem states that if bounded analytic in the unit disc Δ functions f_j satisfy $\sum_{j=1}^N |f_j|^2 \geq \delta^2 > 0$, then there exist bounded analytic functions $g_j \in H^\infty(\Delta)$ such, that $\sum_{j=1}^N g_j f_j = 1$. This is equivalent to the fact that the unit disk Δ is dense in the maximal ideal space of the algebra H^∞ of bounded analytic functions, but the importance of the Corona Theorem goes much beyond the theory of maximal ideals of H^∞.

  It turned out that the Corona Theorem, and especially its generalization, the so called Matrix (Operator) Corona Theorem, play an

important role in operator theory (such as the angles between invariant subspaces, unconditionally convergent spectral decompositions, computation of spectrum, etc.).

In this talk I am going to discuss some new results related to the Corona Problem, describe connections with operator theory and with the geometry of holomorphic vector bundles, as well as state some new (and some old) open problems.

It worth mentioning that in the area "around corona" there are much more open questions than answers. In particular, it is still an open problem to find out if the Corona Theorem holds for a polydisc or a unit ball in $\mathbb{C}^n$ or for general domain in $\mathbb{C}$.

- **Sergei Treil (Brown University)**
  "Two weight estimates and perturbations of self-adjoint operators".
  Dia i hora: 15 de juny, a les 12.00 h.
  Lloc: aula gran del CRM.
  Abstract: In the talk I am going to present a special regularization of the Cauchy integral, appearing in the theory of rank one perturbations of self-adjoint operators. I will show how the interplay between operator theory and harmonic analysis allows to obtain new results in both areas. In particular, as a corollary of an elementary theory of two weight estimates I will get a new result in perturbation theory, namely, a sufficient condition of the disappearance of the singular spectrum.

- **Ciprian Demeter (UCLA)**
  "Breaking the duality in the Return Times Theorem".
  Dia i hora: 19 de juny, a les 15.00 h.
  Lloc: aula T2 de la UB.
  Abstract: Bourgain proved the following Return Times theorem: Let $\mathbf{X} = (X, \Sigma, \mu, \tau)$ be a dynamical system. Let also $1 \leq p, q \leq \infty$ be such that $\frac{1}{p} + \frac{1}{q} \leq 1$. For each function $f \in L^p(X)$ there is a universal set $X_0 \subseteq X$ with $\mu(X_0) = 1$, such that for each second dynamical system $\mathbf{Y} = (Y, F, \nu, \sigma)$, each $g \in L^q(Y)$ and each $x \in X_0$, the averages

$$\frac{1}{N} \sum_{n=0}^{N-1} f(\tau^n x) g(\sigma^n y)$$

converge ν- almost everywhere. We show how to break the duality in this theorem. More precisely, we prove that the result remains true if $p > 1$ and $q \geq 2$. We emphasize the strong connections between this result and the Carleson-Hunt theorem on the convergence of the Fourier series. We also prove similar results for the analog of Bourgain's theorem for series, where no positive results were previously known. This is joint work with Michael Lacey, Terence Tao

and Christoph Thiele.

- **Nicola Arcozzi (Universitat de Bolonya)**
"Multipliers of the Drury-Arveson space".
Dia i hora: 23 de juny, a les 10.30 h.
Lloc: aula gran del CRM.

- **Raanan Schul (UCLA)**
"A Traveling Salesman Theorem for 1-Ahlfors-regular sets in a metric space".
Dia i hora: 23 de juny, a les 12 h.
Lloc: aula gran del CRM.
Abstract: I. Hahlomma showed that uniform control of local averages of Menger curvature of a 1-Ahlfors-regular set K, allow one to obtain a 1-Ahlfors-regular curve containing K. We show the converse to this theorem, i.e. that for a 1-Ahlfors-regular curve we have uniform control of local averages of Menger curvature. All of this is done in the setting of an abstract metric space. Thus giving a characterization of 1-Ahlfors-regular subset of 1-Ahlfors-regular curves in a metric space.

- **Sorina Barza (University of Karlstad)**
"Multivariate Hardy type inequalities on monotone functions".
Dia i hora: 26 de juny, a les 15.00 h.
Lloc: aula T2 de la UB.
Abstract: We present the necessary and sufficient conditions for the boundedness of the multidimensional Hardy operator on Lebesgue spaces restricted to the cone of positive decreasing functions on each variable. We will give some properties of the new Bp classes and present some connections of our results with the normability property of multidimensional analogue of Lorentz spaces. The talk is based on the paper "Hardy's inequalities for monotone functions on partly ordered measure spaces", joint work with N. Arcozzi, J.L. Garcia-Domingo, J. Soria.

- **E.D. Nursultanov (Kazakh Academy of Sciences, Kazakhstan)**
"Net Spaces and Interpolation Methods".
Dia i hora: 26 de juny, a les 16.00 h.
Lloc: aula T2 de la UB.
Abstract: In this paper the applications of the net spaces in the different problems are demonstrated. The upper and lower estimates of the norm of a function from L^p are obtained in terms of the Fourier coefficients. We also study the upper and lower estimates of the norm of the convolution in L^p. Moreover, Fourier series in Lorentz

spaces and multiplier problem are investigated.

- **Marianna Csörnyei (University College of London)**
"Structure of null sets, differentiability of Lipschitz functions, and other problems".
Dia i hora: 27 de juny, a les 10.30 h.
Lloc: aula gran del CRM.
Abstract: We describe a decomposition result for Lebesgue negligible sets, and outline some applications to real analysis and geometric measure theory. In particular, we characterise the set of points of non-differentiablity of Lipschitz mappings on $\mathbb{R}^n$. This is a joint work with G. Alberi and D. Preiss.

- **Tony Carbery (University of Edinburgh)**
"Tube nullity and Fourier analysis".
Dia i hora: 27 de juny, a les 12 h.
Lloc: aula gran del CRM.

- **M. Angeles Alfonseca (Kansas State University)**
"Geometric properties of intersection bodies".
Dia i hora: 29 de juny, a les 10.30 h.
Lloc: aula gran del CRM.
Abstract: Intersection bodies are convex bodies whose radial function is a positive definite distribution. They were introduced in 1988 by Lutwak in connection to the Busseman-Petty problem. They also appear as duals of zonoids (convex bodies that can be approximated by polytopes that are sums of segments). In general, no much is known about the geometry of intersection bodies, even of those that are polytopes.

  Recently, Fourier analytic techniques have been introduced in the study of convex bodies, yielding results such as the analytic solution to the Busseman-Petty problem in all dimensions. In 1998, Koldobsky found a necessary condition for a convex body to be an intersection body in terms of the second derivative of its norm. This result allowed him to prove that the unit ball of the q-sum of two spaces X and Y is not an intersection body for finite q.

  In our work we use the techniques of Koldobsky to prove that, in dimension 7 or more, an intersection body cannot be a direct sum of two convex bodies. We also find conditions for a body of revolution that has a face to be an intersection body.

  This is a joint work with D. Ryabogin and A. Zvavitch.

- **Hervé Pajot (Université de Grenoble I)**
"The geometric travelling salesman problem in the Heisenberg group".
Dia i hora: 29 de juny, a les 12 h.

Lloc: aula gran del CRM.

- **Alex Iosevich (University of Missouri, Columbia)**
"Analysis, combinatorics and number theory of the Mattila integral".
Dia i hora: 3 de juliol, a les 15 h.
Lloc: aula petita del CRM.
Abstract: Let P denote a collection of points and L a collection of geometric objects. Counting the number of incidences between the elements of P and L is a classical problem that arises in several areas of mathematics and computer science. We shall refer to the L2 norm, appropriately defined, of this incidence function, as the Mattila integral, introduced by P. Mattila in the study of the Falconer distance problem. We shall discuss analytic, combinatorial and number theoretic aspects of this quantity in Euclidean space and vector spaces over finite fields. Particular emphasis will be placed on number theoretic consequences of analytic inequalities and the restrictions that these connections imply.

- **Guy David (Université de Paris-Sud)**
"Jean Taylor's theorem and minimal sets in $\mathbb{R}^3$".
Dia i hora: 4 de juliol, a les 10.30 h.
Lloc: aula petita del CRM.
Abstract: I shall try to describe a proof of Jaen Taylor's classical theorem that describes minimal (and almost-minimal) sets of dimension 2 in $\mathbb{R}^3$. Such sets are shown to be local C^1 deformations of minimal cones (there are three such cones, that you can see in soap bubbles).

  The motivation for this lecture is potential existence results for closed sets $E \subset \Omega \subset \mathbb{R}^n$ that minimize a functional like

$$J(E) = \int_E h(x) dH^d(x)$$

  under some topological constraints on E , where h is a continuous function on the domain $\Omega \subset \mathbb{R}^n$, with $C^{-1} \leq h \leq C$ on Ω, H^d is the Hausdorff measure (or d-dimensional surface measure), and the constraints also force E to be at least d-dimensional. Think about the Plateau problem for soap films in $\mathbb{R}^3$. Minimizers for J are typical examples of almost-minimal sets.

  A scheme to study such existence problems without currents would use lowersemicontinuity results for the restriction of H^d to almost-minimal sets and a good local description of almost-minimal sets. Such a scheme may well work in some cases, at least when $d = 2$ and $n = 3$, precisely because of Jean Taylor's celebrated regularity result.

If I can, I will concentrate on the proof rather than the motivations.

- **Ignacio Uriarte (Universty of Missouri, Columbia)**
  "Improved Painleve removability for planar quasiregular mappings".
  Dia i hora: 4 de juliol, a les 12 h.
  Lloc: aula petita del CRM.
  Abstract: The classical Painleve problem (characterize geometrically the sets of zero analytic capacity) has been recently solved by Tolsa (with previous partial results by Guy David, etc.). It is natural to try to understand the analogous problem in the quasiconformal world, i.e. understand the removable sets for bounded solutions of the Beltrami equation. I will present the results of a joint work with Astala, Clop, Mateu and Orobitg. It is known that not all compact sets of sigma-finite length in the plane are removable for bounded analytic functions (1 is the critical dimension for this problem). One of our main results is that, somewhat surprisingly, for the analogous quasiconformal problem (removability for bounded K-quasiregular mappings), all sets of sigma-finite measure at the critical dimension are removable. The techniques come from complex analysis and quasiconformal mappings (conformal welding, integral means estimates, Makarov's compression and expansion for conformal mappings), multifractal analysis, nonlinear potential theory (Riesz and Bessel capacities), harmonic analysis (Calderon-Zygmund theory, Hoermander-Mihlin multiplier theorem), geometric measure theory, etc. I will try to make the talk as self-contained as possible.

- **Galia Dafni (Concordia University (Montreal)**
  "Q_α spaces, Carleson-type measures, and capacities".
  Dia i hora: 6 de juliol, a les 10.30 h.
  Lloc: aula petita del CRM.
  Abstract: In joint work with Jie Xiao on duality theorems for Q_α spaces, there arose a natural relationship between the fractional Carleson condition for a measure μ on the upper half-space:

$$\mu(T(B_r)) \leq Cr^d$$

  for all balls B_r in R^n, where $T(B)$ is the tent over B, and a condition involving d-dimensional Hausdorff capacity $\Lambda_d^{(\infty)}$, namely

$$\mu(T(O)) \leq C\Lambda_d^{(\infty)}(O)$$

  for all open sets O in R^n. In recent joint work with Georgi Karadzhov and Jie Xiao, we extend this relationship to more general classes of Carleson-type measures and other capacities based on various function spaces.

- **Sanghyuk Lee (Seoul National University)**
  "On the pointwise convergence of the solutions to Schrödinger equations".
  Dia i hora: 6 de juliol, a les 12 h.
  Lloc: aula petita del CRM.
  Abstract: We improve the known results on the almost everywhere convergence of the solutions to Schrödinger equations in $\mathbb{R}2$. It is to be shown that $\lim_{t\to 0} e^{it\Delta} f = f$ a.e. for all $f \in H^s$ if $s > 3/8$.