Extrapolation: three lectures in Barcelona

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SCHEDULE:
June 18th, 11:00 to 13:00
June 19th, 11:00 to 12:00
PLACE: Centre de Recerca Matemàtica, Room A2 (2nd floor)

CONTENTS:

I hope that these lectures, and the soon to be posted accompanying notes, could be of some interest to graduate students and analysts that would like to see "real time" interactions between soft and hard analysis. A brief outline of the lectures and the basic references follows.

Lecture 1: The K/J method of extrapolation of Jawerth-Milman.

June 18th, 11:00.
Calderón's method [3] for characterizing interpolation inequalities via rearrangement inequalities is extended via the use of inequalities between the K and J functionals and the strong fundamental lemma of interpolation theory [5]. In particular, this allows for a complete characterization of inequalities with very general decays at the end points of an interpolation scale and thus plays a fundamental role in the theory of [9].
To my mind this is one of the most interesting results of [9] but the one that probably has received the least amount of attention!
I will start with the classical context of Calderón's theory before moving to the more general point of view of [9] and further illustrate these ideas with a general bilinear interpolation/extrapolation theorem obtained in [10].

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Lecture 2: Sobolev inequalities and a new extrapolation theorem.

June 18th, 12:00.

One of the main applications of the classical Yano type extrapolation theorems in the 60's were the end point Sobolev inequalities of Trudinger [16], Strichartz [15], and many others. The usual methods here involve: i) to prove end point inequalities (e.g. Gagliardo-Nirenberg, etc), ii) to interpolate those inequalities, keeping track of the constants and finally (iii) to "extrapolate" end point results. However, it turns out that Sobolev inequalities need not be interpolated or for that matter extrapolated: rather one suitably chosen end point inequality can be extrapolated to prove ALL Sobolev inequalities!! With Joaquim Martin and Evgeniy Pustylnik [14] we have shown that from one Sobolev inequality one can prove (in the spirit of the Calderón idea discussed in the first Lecture) a pointwise rearrangement inequality for the gradient that implies "all" the Sobolev inequalities including all the extrapolations of such! In this lecture I will show a slightly more general abstract principle by Cwikel-Jawerth-Milman [6] (cf. also Martin-Milman [11]) that explains how this can be applied to other operators/scales with suitable cancellations (e.g. square functions) yielding a new approach to certain martingale inequalities of Burkholder-Gundy [2], Herz [8] and others.

Lecture 3 : A new extrapolation theorem of Maz'ya-Rubio type.

June 19th, 11:00.

With Joaquim Martin (cf. [12]) we developed a connection between the Δ and $\sum$ methods of extrapolation of Jawerth-Milman [9] and Rubio de Francia's celebrated extrapolation theorem [7]. In more recent joint work with Joaquim Martin (cf. [13]) we have shown a new extrapolation theorem of "Maz'ya-Rubio" type that allows to characterize all the weighted Sobolev norm estimates from a suitable initial estimate.

If there is time, or at the end of the lecture notes, I will at least briefly indicate some intriguing connections of extrapolation with ideals of operators (via the recent work of Astashkin-Lykov [1]) and with the work of Corach and his collaborators on differential geometry of operators and interpolation theory (cf. [4]).

References

[1] S. Astashkin and Lykov, *Extrapolation description of rearrangement invariant spaces and related problems*, in Banach and Function Spaces III, Yokahama, 2010.

[2] D. L. Burkholder and R. F. Gundy, *Extrapolation and interpolation of quasi-linear operators on martingales*, Acta Math. **124** (1970), 249-304.[BG] Burkholder, Gundy

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[4] E. Andruchow, G. Corach, M. Milman and D. Stojanoff, *Geodesics and interpolation*, Rev. Un. Mat. Argentina 40 (1997), no. 3-4, 83-91.

[5] M. Cwikel, B. Jawerth and M. Milman, *On the fundamental lemma of interpolation theory*, J. Approx. Th. 60 (1990), 70-82.

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[8] C. Herz, *An interpolation principle for martingale inequalities*, J. Funct. Anal. **22** (1976), 1-7.

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[11] J. Martin and M. Milman, *Sobolev inequalities, rearrangements, isoperimetry and interpolation spaces*, Contemporary Mathematics 545 (2011), pp 167-193.

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[15] R. S. Strichartz, *A note on Trudinger's extension of Sobolev's inequality*, Indiana Univ. Math. J. 21 (1972), 841-842.

[16] N. Trudinger, *On imbeddings into Orlicz spaces and some applications*, J. Math. Mech. 17 (1967), 473-483.